HAYFIELD CROSS
Church of England
School

Calculation Policy
January 2024

Addition

Skill	Year Group(s)	Representations and models
Add two 1-digit numbers to 10	1	Objects, Numicon, tens frames (within 10), part-whole model, number line (labelled).
Add 1 and 2-digit numbers to 20	1/2	Objects, Numicon, tens frames (within 20), part-whole model, number line (labelled), straws.
Add 3 1-digit numbers	2	Part-whole model, bar model, tens frames (within 20), jottings.
Add 1 and 2-digit numbers to 100.	2/3	Part-whole model, bar model, number lines (labelled), hundred square, Base 10, jottings.
Add two 2-digit numbers to 100.	2/3	Part-whole model, bar model, number lines (labelled), hundred square, jottings.
Add numbers with up to 3-digits	3	Part-whole model, bar model, Base 10, place value counters, column addition.
Add numbers with up to 4-digits	4	Part-whole model, bar model, Base 10, place value counters, column addition.
Add numbers with more than 4-digits.	5/6	Part-whole model, bar model, place value counters, column addition.
Add numbers with up to 3 decimal places.	5/6	Part-whole model, bar model, place value counters, column addition.

Addition

CONCRETE (objects)

AUGMENTATION STRUCTURE —counting on and increasing

Start with the larger number on the bead string and then add on the smaller number 1 by 1 to find the total.

Count aloud.

PICTORIAL

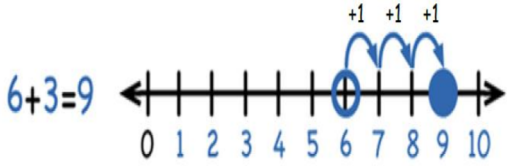
AUGMENTATION STRUCTURE: - Counting on and increasing

This phase can be supported alongside the concrete phase, using cubes or bead strings to show the connection to the number line.

The children have a starting point (which they do not need to count as with the aggregation structure). They should now 'count on' along the number line.

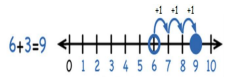
Children put the largest number into their heads and then count on from that point either by physically moving themselves along a larger number line or by using their finger on a smaller one.

Focus more on the actions and language used to link the picture (number line) to their language of what is happening.



ABSTRACT a

Children should now use the **concrete** or **pictorial** phases to develop the link with making mathematical statements from what they are doing/seeing and what **language** they are using.

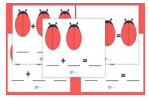


E.g. Six and three more equals nine.

6 + 3 = 9

Start by using numeral and symbol cards that can be manipulated into forming mathematical statements.

Then move on to children filling in writing frames by writing in the numerals and symbols into empty boxes.



Finally, children should be able to compose mathematical statements of their own to demonstrate what has happened.

5 + 4 = 9

ABSTRACT b

Place the larger number in your head and count on the smaller number to find the total.

Skill: Adding 1-digit numbers to 10

Year 1

Addition

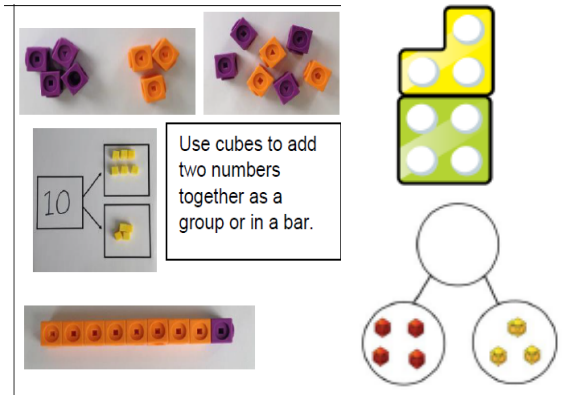
CONCRETE (objects)

AGGREGATION STRUCTURE—the union of two sets

Use two sets of objects, join them altogether and count to find the total.

PART-WHOLE MODEL

Use a range of objects to model this and make it cross curricular where possible.

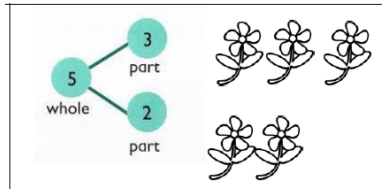


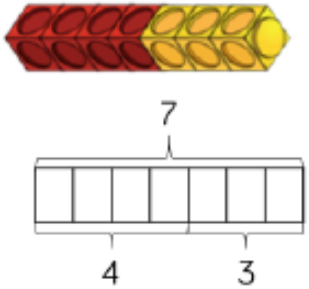
PICTORIAL

AGGREGATION STRUCTURE—the union of two sets

Use pictures to add two numbers together as a group or in a bar.

PART-WHOLE MODEL





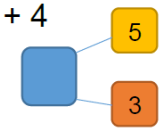
ABSTRACT

AGGREGATION STRUCTURE—the union of two sets

Use part-whole models to move into abstract and to introduce missing number problems and use language to discuss mathematical statements and symbols.

$4 + 3 = 7$

$10 = 6 + 4$



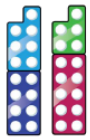
Skill: Adding 1-digit numbers to 10

Addition

When adding 1-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. It is best to ensure this concept is fully understood in the concrete phase before moving into the pictorial or abstract phase.

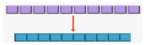
CONCRETE (objects)

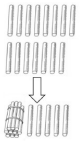
Use two sets of objects, join them altogether and count to find the total—**PART-WHOLE MODEL**
Use a range of objects to model this and make it cross curricular where possible.



In Year 2, the focus will be on exchanging groups ten of ones for a ten 'stick' or 'bundle'.

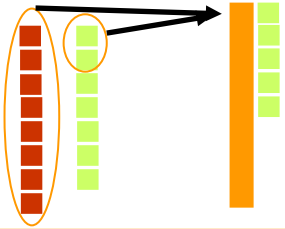
ten ones





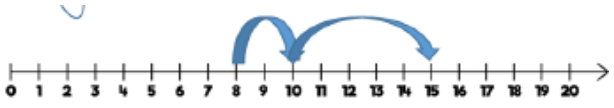
one group of 10

The children will also become familiar with using the physical Base 10 (tens 'sticks' and ones).



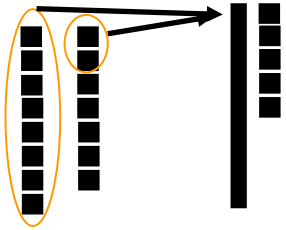
PICTORIAL

In Year 1, children use a number line or draw Numicon jottings to help them solve these calculations when using the pictorial approach.



In Year 2, children will use Base 10 jottings to support them to solve the calculation.

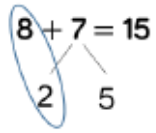
For example, **8 + 7 =**

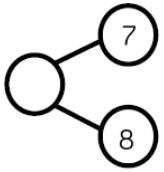


ABSTRACT

Children will use part-whole models to move into abstract and to introduce missing number problems and use language to discuss mathematical statements and symbols.

8 + 7=15





Skill: Add 1 and 2-digit numbers to 20

Year 1/2

Addition

When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10. This supports children in their understanding of commutativity.

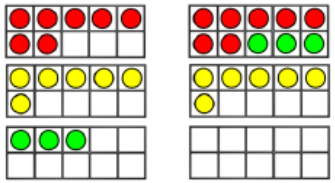
CONCRETE (objects)

ADDING THREE SINGLE DIGITS

3 + 7 + 6 = 17

Look for number bonds to 10 and then see what is leftover.

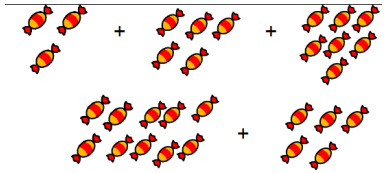
Use counters on a 10s frame and then move them altogether to aid counting and identifying number bonds.



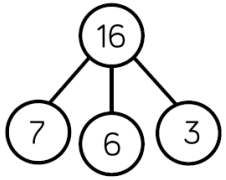
PICTORIAL

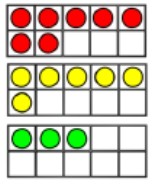
ADDING THREE SINGLE DIGITS

Draw a picture to recombine the objects to make 10 plus how many leftover to create a more efficient statement.



Use a part-whole model to represent the numbers, then draw counters in 10s frames to solve the calculation.

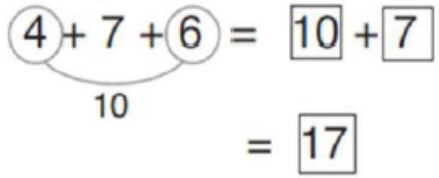




ABSTRACT

ADDING THREE SINGLE DIGITS

Combine the numbers that make ten (where possible) and add on what is leftover mentally.



Skill: Add three 1-digit numbers

Year 2

Addition

Base 10 and place value counters are the most effective resources when adding numbers with 3-digits.

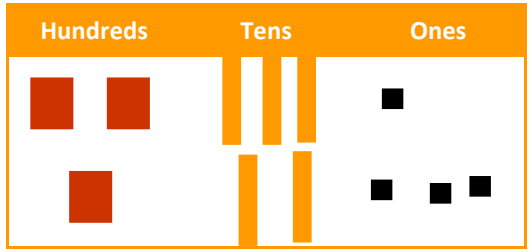
CONCRETE (objects)

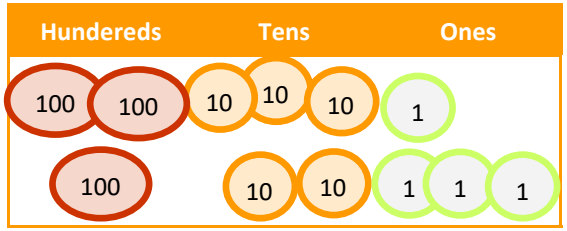
COLUMN METHOD—No bridging

Add together the ones first and then the tens and then the hundreds.

Use the Base 10 blocks first before moving onto place value counters.

231 + 123 = 354



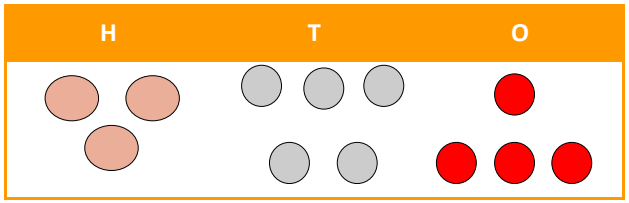


PICTORIAL

COLUMN METHOD—No bridging

After practically using the Base 10 and counters, children can draw the counters to help them to solve addition number statements.

231 + 123 = 354



ABSTRACT part 1

COLUMN METHOD—No bridging

Children to use the **expanded column method** to add the ones, tens and then hundreds.

200 and 30 and 1

100 and 20 and 3

_____

300 and 50 and 4

_____

ABSTRACT part 2

Children to use the **formal column method** to add the ones, tens and then hundreds.

231

123

_____

354

Skill: Add numbers with up to 3-digits.

Year 3

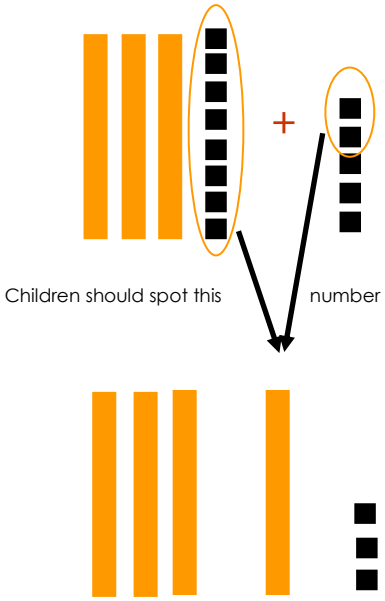
Addition

When adding single digits to a 2-digit number, children should count on from the larger number. They should also use their knowledge of number bonds to support their addition.

CONCRETE (objects)

Children to use Base 10 to add a 1-digit and a 2-digit number within 100.

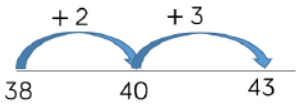
38 + 5 = 43



Children should spot this number bond to 10. 8 + 2 = 10

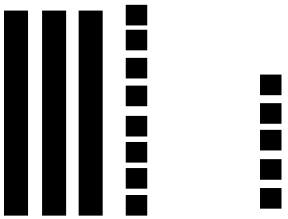
PICTORIAL

Children will begin by using a number line to count on from the greatest number.



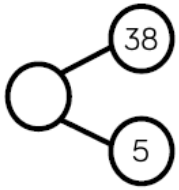
Children will move away from using the physical Base 10 to drawing visual representations.

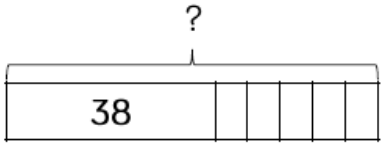
38 + 5 = 43



ABSTRACT

Children will begin to count up in their heads to solve the calculation. They could picture a Part-Whole model, a number line or a bar model in their heads.





Skill: Add 1 and 2-digit numbers to 100.

Year 2/3

Addition

In Year 2, pupils will become efficient at using Base 10 and Base 10 jottings to add two 2-digit numbers together.

CONCRETE (objects)

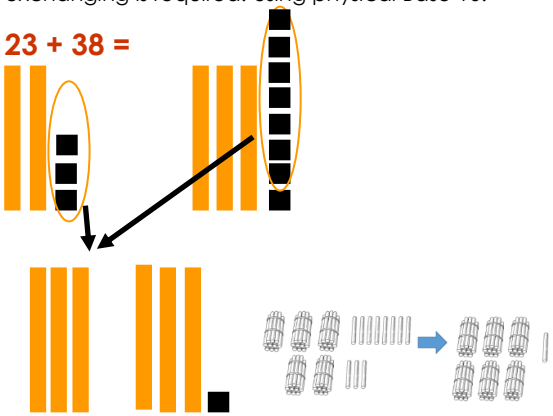
Without bridging

This is when the calculation does not bridge 10 and therefore no exchanging is required. Using physical Base 10. **2 + 33 =**

With bridging

This is when the calculation bridges 10 and therefore exchanging is required. Using physical Base 10.

23 + 38 =



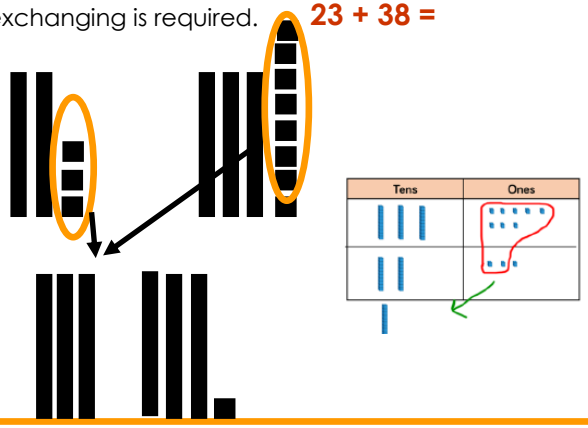
PICTORIAL

Without bridging

This is when the calculation does not bridge 10 and therefore no exchanging is required. Using Base 10 jottings. **24 + 33 =**

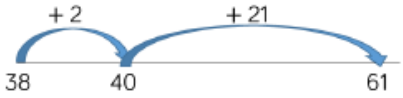
With bridging

This is when the calculation bridges 10 and therefore exchanging is required. **23 + 38 =**



ABSTRACT

Children can use a blank number line to count on to find the total. Children should be encouraged to jump to multiples of 10 to become more efficient.



Skill: Add two 2-digit numbers to 100.

Year 2

Addition

In Year 3, pupils are encouraged to use the formal column method alongside Base 10 or place value counters until they fully understand and can explain the process. As numbers become larger, Base 10 and place value counters become less efficient.

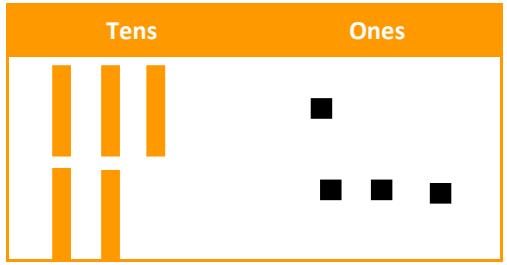
CONCRETE (objects)

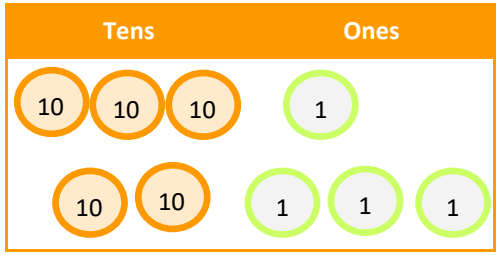
COLUMN METHOD—No Bridging

Add together the ones first and then the tens.

Use Base 10 before moving onto place value counters.

31 + 23 = 54



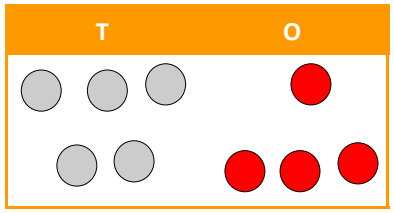


PICTORIAL

COLUMN METHOD—No bridging

After practically using the Base 10 and counters, children can draw counters in a place value grid to help them to solve addition number statements.

31 + 23 = 54



ABSTRACT part 1

COLUMN METHOD—No bridging

Children to use the **expanded column method** to add the units/ones and then the tens.

30 and 21

20 and 3

_____

50 and 4

_____

ABSTRACT part 2

COLUMN METHOD—NO REGROUPING (no carrying down)

Children to use the **formal column method** to add the units/ones and then the tens.

31

23

_____

54

Skill: Add two 2-digit numbers to 100.

Year 3

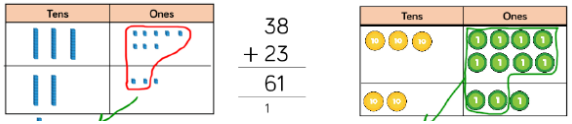
Addition

In Year 3, pupils are encouraged to use the formal column method alongside Base 10 or place value counters until they fully understand and can explain the process. As numbers become larger, Base 10 and place value counters become less efficient.

CONCRETE (objects)

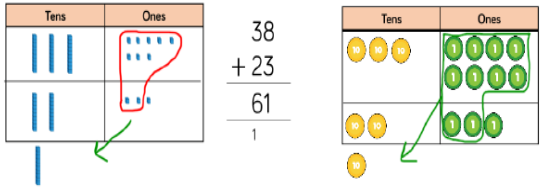
COLUMN METHOD—Bridging

Make both numbers on a place value grid.



Add the ones and exchange ten ones for one tens stick.

Add up the rest of the columns, until every column has been added.



Place value counters can also be used to show this.

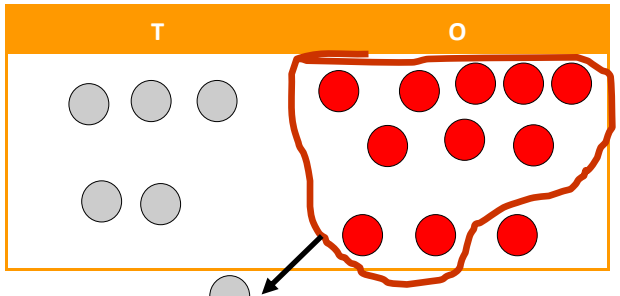
PICTORIAL

COLUMN METHOD—Bridging

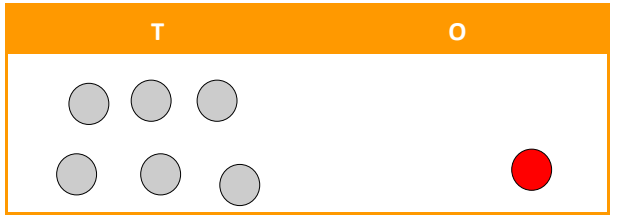
Children can draw a pictorial representation of the columns and objects to further support their understanding.

This is a quicker and more efficient way than using the counters ***if they are secure with what the exchanging means.***

38 + 23 = 61



Exchange these ones for a 10.



ABSTRACT part 1

COLUMN METHOD—Bridging

Start by partitioning the numbers before moving on to clearly show the exchanging.

20 + 5

40 + 8

_____

70 + 3

_____

10

ABSTRACT part 2

Children to use the **formal column method** to add the ones and then the tens, showing exchanging.

25

48

_____

73

_____

Skill: Add two 2-digit numbers to 100.

Year 3

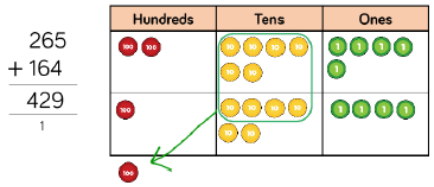
Addition

Base 10 and place value counters are the most effective practical resources when adding numbers with 3-digits.

CONCRETE (objects)

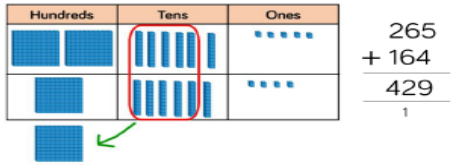
COLUMN METHOD— Bridging

Make both numbers on a place value grid.



Add the ones and exchange ten ones for one ten if needed. Then repeat for the tens and hundreds columns.

Add up the rest of the columns, until every column has been added.



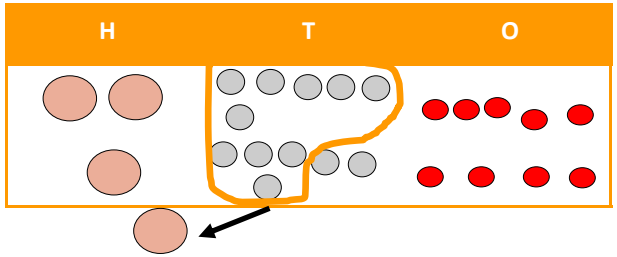
Base ten can also be used to show this.

PICTORIAL

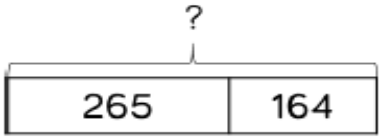
COLUMN METHOD—Bridging

After practically using Base 10 and counters, children can draw the counters to help them to solve addition number statements.

265 + 164 = 429



It could also be represented in a bar model.



ABSTRACT

COLUMN METHOD—Bridging

The children would then move onto using the column addition without any physical objects or pictorial representations.

265
+ 164
429
1

Skill: Add numbers with up to 3-digits.

Year 3

Addition

The children will begin adding numbers with up to 4-digits with calculations that contain no bridging. They will follow the same methods as in Year 3 but with a thousands column. After this, they will move onto bridging the ones, tens, hundreds or thousands.

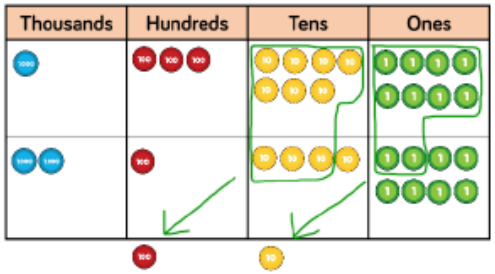
CONCRETE (objects)

COLUMN METHOD— Bridging

Make both numbers on a place value grid.

Add the ones and exchange ten ones for one ten if needed. Then repeat for the tens, hundreds and thousands columns.

Add up the rest of the columns, until every column has been added.

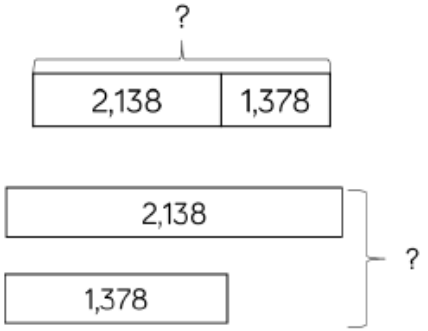


PICTORIAL

COLUMN METHOD— Bridging

After practically using the Base 10 and counters, children can draw the counters (jottings) to help them to solve addition number statements. See the previous page (Year 3) for a similar example. The place value grid would have a thousands column.

The children can also represent, and solve, their calculation using a bar model. They could then add the ones, tens, hundreds and thousands together using jottings (as mentioned above).



ABSTRACT

COLUMN METHOD— Bridging

The children then move onto using the column addition without any physical objects or pictorial representations.

	1	3	7	8
+	2	1	4	8
	3	5	2	6
		1	1	

Skill: Add with up to 4-digits

Year 4

Addition

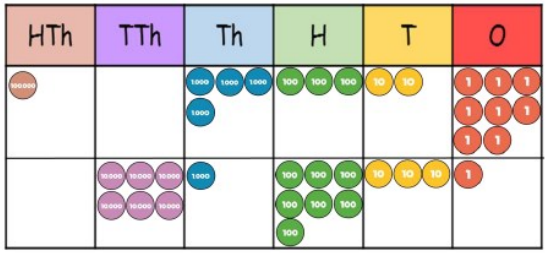
At this stage, children should be encouraged to use the concrete and pictorial method a handful of times so that they can visually see the process and then they should be encouraged to work in the abstract. This is because using the column method is the most efficient when adding larger numbers.

CONCRETE (objects)

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

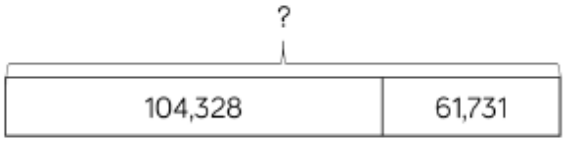
For example,

104,328 + 61,731 = 166,059

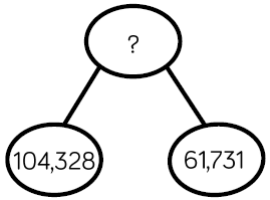


PICTORIAL

Children can draw a bar model to help them visualise the calculation that they are solving.



They could also draw a part-whole model.



ABSTRACT

The children should be encouraged to use the column method. They will need to ensure that they lay out their calculations in the correct columns to avoid any mistakes.

1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

1

Skill: Add with more than 4-digits.

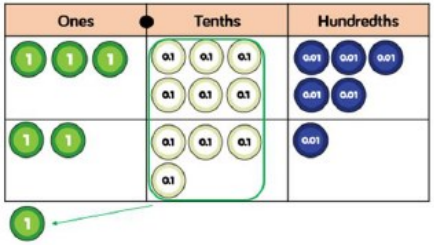
Years 5 & 6

Addition

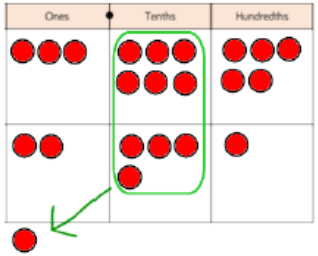
Place value counters on a place value grid are the most effective resources when adding numbers with 1, 2 and then 3 decimal places. Children must have experiences of adding decimals in a variety of contexts including measures and money.

CONCRETE (objects)

Children should begin by adding numbers with one decimal place, then 2 and 3 decimal places.

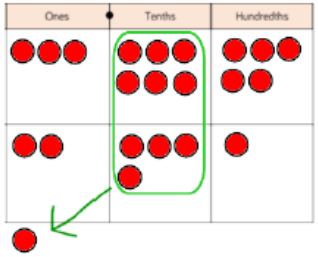


As well as place value counters, children could also use plain counters. This makes the transition to pictorial more seamless.

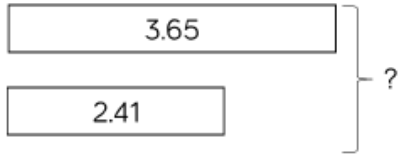


PICTORIAL

Children can begin by drawing place value grids and counters to support their developing calculation skills.



Children will then move onto drawing a bar model to help them visualise the calculation.



ABSTRACT

Once children understand the concept of adding decimals, then they can use the column method.

$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

It is important that they ensure the decimal points are in line, regardless of how many 'numbers' are before/after the decimal point.

Skill: Add with up to 3 decimal places.

Years 5 & 6

Vocabulary: Addi–

KEY VOCABULARY YEAR 1

Number / Digit (0-9) / Numerals

Zero, one, two, three (to 100)

None / Nothing

Count (on/up/to/from)

What comes after….?

More, many, greater than

Equal to, the same as, (including equals sign)

Number bonds, number line

Operation / Symbol / Sign / +

Add, more, plus, make, total, altogether

Inverse

Partition

Difference between

How many more to make…?

Near doubles

Equal to/the same as/total

KEY VOCABULARY YEAR 2

See Year 1 Vocabulary plus the following Y2 specific vocabulary:

Numerals/numbers

Numbers in numerals and words (one - one hundred)

Greater than,

Larger than

How much/many altogether?

How many more is …. than ….?

How many more do you need to make …?

Count on/forwards

Partition

Exchange/bridging ten

Equal to/the same as/total

Altogether

KEY VOCABULARY YEAR 3

See Year 2 Vocabulary plus the following Y3 specific vocabulary:

Column addition

Hundreds boundary

KEY VOCABULARY YEAR 4

See Year 3 Vocabulary plus the following Year 4 specific vocabulary:

Inverse

KEY VOCABULARY YEARS 5 & 6

See Year 4 Vocabulary plus the following Year 5 specific vocabulary:

Ones boundary,

Tenths boundary

Subtraction

Skill	Year Group(s)	Representations and models
Subtract two 1-digit numbers within 10	1	Objects, Numicon, tens frames (within 10), part-whole model, number line (labelled), bead strings (10)
Subtract 1 and 2-digit numbers to 20.	1	Objects, Numicon, tens frames (within 20), part-whole model, number line (labelled), straws.
Subtract 1 and 2-digit numbers to 100	2	Part-whole model, bar model, number lines (labelled), straws, jottings, Base 10.
Subtract two 2-digit numbers.	2 & 3	Part-whole model, bar model, number lines (blank), hundred square, Base 10, jottings, bar model, column subtraction.
Subtract numbers with up to 3-digits.	3	Part-whole model, bar model, Base 10, place value counters, column subtraction.
Subtract numbers with up to 4-digits.	4	Part-whole model, bar model, Base 10, place value counters, column subtraction.
Subtract numbers with more than 4-digits.	5	Part-whole model, bar model, Base 10, place value counters, column subtraction.
Subtract numbers with to 3 decimal places.	5 & 6	Part-whole model, bar model, place value counters, column subtraction.

Subtraction

Part-whole models, tens frames, bar models and Numicon support the partitioning of numbers.
Tens frames, number lines and single bar models support reduction.

CONCRETE (objects)

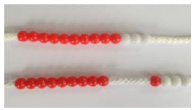
Partitioning

Children can use Numicon to partition the number they are subtracting.

7—3 =

Reduction

This is when the children 'take away' an amount. Make the larger number with the beads and then slide the beads along as you count back in ones.



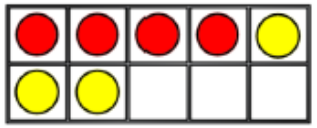
Alternatively, use counters and move them away from the group as you take away, counting backwards out loud as you do.

PICTORIAL

Partitioning

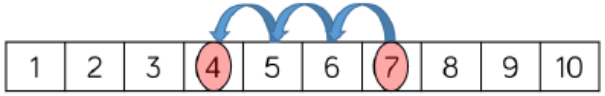
The next step is to move onto using a tens frame or a labelled number line.

With a tens frame, you draw the greatest number and subtract (cross out) the smallest number.



Reduction

With a number line, you start on the greatest number and jump backwards by the amount you are subtracting.



ABSTRACT

Partitioning

Children confident in counting back mentally in ones by recognising symbols and reading mathematical statements:

E.g. **18—6 = 12**

8—5 = 3

Reduction

Children confidently count back mentally.

Children may use their fingers for visual support.

Skill: Subtract two 1-digit numbers within 10.

Year 1

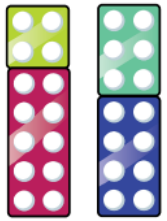
Subtraction

When subtracting 1-digit numbers that cross 10, it is important to highlight the importance of one ten equalling ten ones.

CONCRETE (objects)

Partitioning

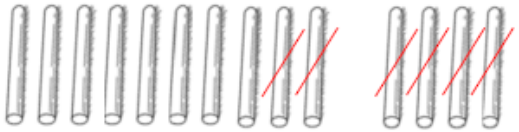
Children can use Numicon to partition the number they are subtracting.



14—6 = 8

Reduction

Use counters or straws and move them away from the group as you take away, counting backwards out loud as you do.

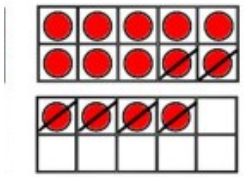


PICTORIAL

Partitioning

The next step is to move onto using a tens frame or a labelled number line.

With two tens frame, you draw the greatest number and subtract (cross out) the smallest number.



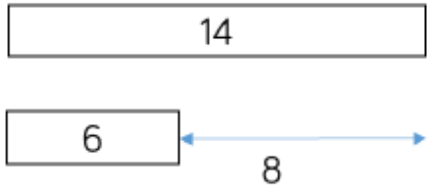
Reduction

With a number line, you start on the greatest number and jump backwards by the amount you are subtracting.

ABSTRACT

Children use their knowledge to solve word problems based on finding the difference or comparing two numbers.

They may find is useful to picture a bar model to visualise the calculation.



Skill: Subtract 1 and 2-digit numbers to 20.

Year 1

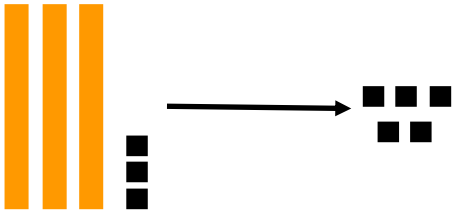
Subtraction

At this stage children will be using Base 10 to support their calculation skills. As numbers become larger straws become much less efficient. Children should be encouraged to count in multiples of 10 to become more efficient.

CONCRETE (objects)

Children to use Base 10 to subtract a 1-digit and a 2-digit number within 100.

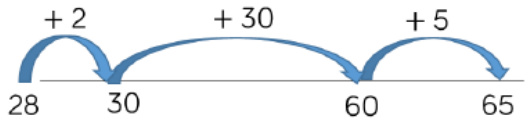
38 - 5 = 33



Children physically move the Base 10 away from the group so that they can see that the number is removed. Then the children count how many are left.

PICTORIAL

Children will begin by using a number line to count on from the smallest number.

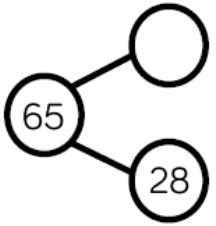


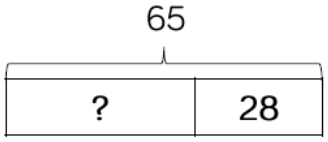
Children will move away from using the physical Base 10 to drawing visual representations. They cross out the number that they need to subtract.

38 - 5 = 33

ABSTRACT

Children will begin to count back in their heads to solve the calculation. They could visualise a Part-Whole model, a number line or a bar model in their heads.





Skill: Subtract 1 and 2-digit numbers to 100

Year 2

Subtraction

Children should learn how to complete the column method in the concrete stage. This skill should be really embedded before moving onto pictorial and abstract methods.

CONCRETE (objects)

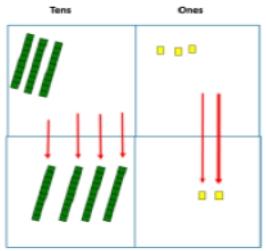
COLUMN METHOD WITHOUT REGROUPING

Use Base 10 to physically make the larger number and then take away the smallest number.

Count what is leftover using tens and units knowledge.

E.g.

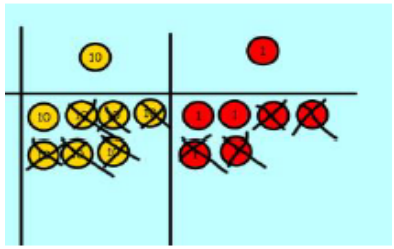
75—42 = 33



PICTORIAL

COLUMN METHOD WITHOUT REGROUPING

Draw the Base 10 or Place Value counters and cross them out to take away.



ABSTRACT

COLUMN METHOD WITHOUT REGROUPING

Partition the numbers to show place value and use the expanded column method.

40 and 5

\-

20 and 3

_____

20 and 2

_____

This will lead to a more formal column method when knowledge of place value is secure.

45

\-

23

_____

22

_____

Skill: Subtract two 2-digit numbers to 100.

Years 2 & 3

Subtraction

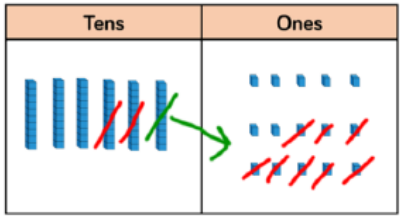
Children should learn how to complete the column method in the concrete stage. This skill should be really embedded before moving onto pictorial and abstract methods.

CONCRETE (objects)

COLUMN METHOD WITH REGROUPING

Use Base 10 to start with before moving on to place value counters. Start with one exchange.

Always start with the units/ones



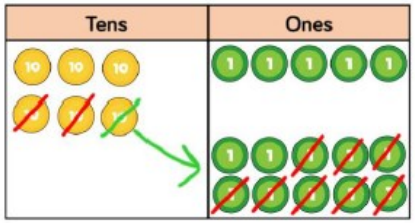
$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}5 \\ -\ 28 \\ \hline 37 \end{array}$$

PICTORIAL

COLUMN METHOD WITH REGROUPING

Draw Base 10 or place value counters onto a place value grid and show what you have taken away by crossing out as well as showing the exchanges you have made.

$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}5 \\ -\ 28 \\ \hline 37 \end{array}$$



ABSTRACT

COLUMN METHOD WITH REGROUPING

Partition the numbers to show place value and use the expanded column method to show exchanging by crossing out the original numbers.

$$\begin{array}{r} 60 + 5 \\ -\ 20 + 8 \\ \hline 50 + 15 \\ -\ 20 + \ 8 \\ \hline 30 + 7 \end{array}$$

Children are to use the **formal column method** to subtract the ones and then the tens, showing exchanging.

$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}5 \\ -\ 28 \\ \hline 37 \end{array}$$

Skill: Subtract two 2-digit numbers to 100.

Years 2 & 3

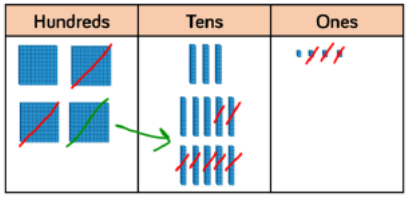
Subtraction

Base 10 and place value counters are the most effective resource when subtracting larger numbers. It is important to ensure that children write out their calculation alongside any concrete resources so that they can see the link between the concrete and abstract methods.

CONCRETE (objects)

Children begin by using Base 10 to practically solve the subtraction calculations. They should write the column method calculation alongside the physical resources.

435—273 = 262



$$\begin{array}{r} {}^{3}\,{}^{1}4\,3\,5 \\ -\ 2\,7\,3 \\ \hline 2\,6\,2 \end{array}$$

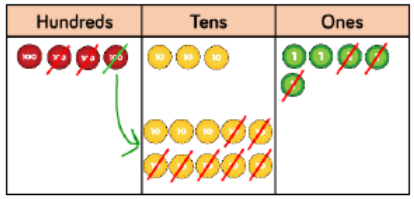
Children begin by making the largest (first) number. Then they move out the number of ones, tens and then hundreds that they are subtracting. Exchanging is to occur as and when it is needed.

PICTORIAL

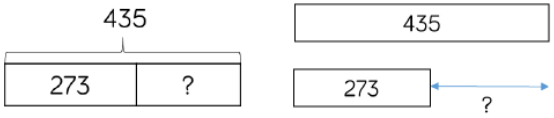
After practically using Base 10 and counters, children can draw the counters to help them to solve subtraction number statements.

435—273 = 262

$$\begin{array}{r} {}^{3}\,{}^{1}4\,3\,5 \\ -\ 2\,7\,3 \\ \hline 2\,6\,2 \end{array}$$



It could also be represented in a bar model.



ABSTRACT

The children then move onto using the column addition without any physical objects or pictorial representations.

$$\begin{array}{r} {}^{3}\,{}^{1}4\,3\,5 \\ -\ 2\,7\,3 \\ \hline 2\,6\,2 \end{array}$$

Skill: Subtract numbers with up to 3-digits.

Year 3

Subtraction

The children will subtract numbers with up to 4-digits with calculations that contain no bridging. They will follow the same methods as in Year 3 but with a thousands column. After this, they will move onto bridging the ones, tens, hundreds or thousands.

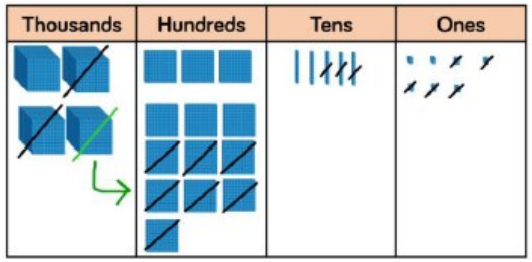
CONCRETE (objects)

Make the largest number on a place value grid.

Subtract the ones and exchange ten ones for one ten if needed. Then repeat for the tens, hundreds and thousands columns.

Add up the remaining pieces to Base 10 to get the answer.

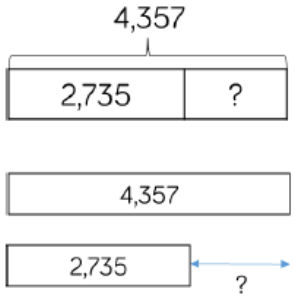
4,357—2,735 =1,622



PICTORIAL

After practically using Base 10 and counters, children can then draw the counters (jottings) to help them to solve addition number statements. See the previous page (Year 3) for a similar example. The place value grid would have a thousands column.

The children can also represent, and solve, their calculation using a bar model. They could then subtract the ones, tens, hundreds and thousands together using jottings (as mentioned above).



ABSTRACT

The children then move onto using the column subtraction without any physical objects or pictorial representations.

$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{3}57 \\ -\ 2735 \\ \hline 1622 \end{array}$$

Skill: Subtract with up to 4-digits.

Year 4

Subtraction

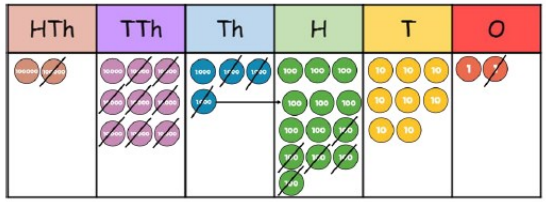
At this stage, children should be encouraged to use the concrete and pictorial method a handful of times so that they can visually see the process and then they should be encouraged to work in the abstract. This is because using the column method is the most efficient when subtracting larger numbers.

CONCRETE (objects)

Place value counters or plain counters on a place value grid are the most effective concrete resources when subtracting numbers with more than 4 digits.

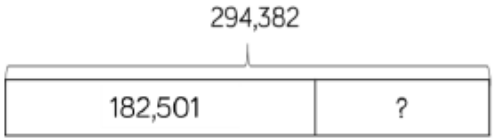
For example,

294,382—182,501 = 11,881

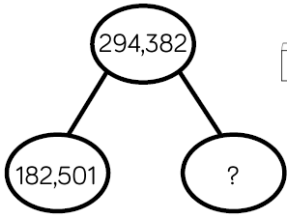


PICTORIAL

Children can draw a bar model to help them visualise the calculation that they are solving.



They could also draw a part-whole model.



ABSTRACT

The children should be encouraged to use the column method. They will need to ensure that they lay out their calculations in the correct columns to avoid any mistakes.

	2	9	$\cancel{3}^{2}$	1_{3}	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

Skill: Subtract numbers with more than 4-digits.

Years 5 & 6

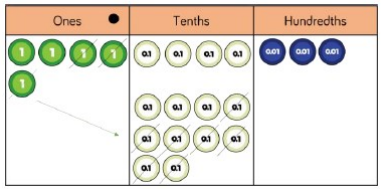
Subtraction

Place value counters on a place value grid are the most effective resources when subtracting numbers with 1, 2 and then 3 decimal places. Children must have experiences of adding decimals in a variety of contexts including measures and money.

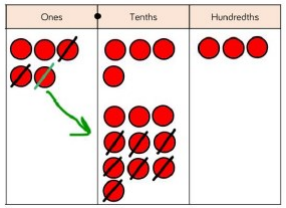
CONCRETE (objects)

Children should begin by subtracting numbers with one decimal place, then 2 and 3 decimal places.

5.43—2.7 = 2.73



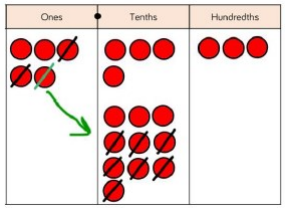
As well as place value counters, children could also use plain counters. This makes the transition to pictorial more seamless.



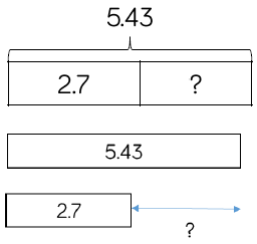
PICTORIAL

Children can begin by drawing place value grids and counters to support their developing calculation skills.

5.43—2.7 = 2.73



Children will then move onto drawing a bar model to help them visualise the calculation.



ABSTRACT

Once children understand the concept of subtracting decimals, then they can use the column method.

$$\begin{array}{r} {}^{4}\not{5}.{}^{1}43 \\ -\ 2.7 \\ \hline 2.73 \end{array}$$

It is important that they ensure the decimal points are in line, regardless of how many 'numbers' are before/after the decimal point.

Skill: Subtract numbers with to 3 decimal places.

Year 5 & 6

Vocabulary: Sub–

KEY VOCABULARY YEAR 1

Number / Digit (0-9) / Numerals

Zero, one, two, three (to 100)

None / Nothing

Count (down/back from)

What comes before….?

Subtract/take away

How many are let/left over?

How many have gone?

One less, two less, ten less

How many fewer is …. Than….?

Difference between

Equals/is the same as

Number bonds

Missing number

partitioning

KEY VOCABULARY YEAR 2

See Year 1 Vocabulary plus the following Y2 specific vocabulary:

Base 10

Jottings

Once hundred less

Number facts

Exchanging

Regrouping

partitioning

Tens boundary

KEY VOCABULARY YEAR 3

See Year 2 Vocabulary plus the following Y3 specific vocabulary:

Column subtraction

Hundreds boundary

KEY VOCABULARY YEAR 4

See Year 3 Vocabulary plus the following Year 4 specific vocabulary:

Inverse

KEY VOCABULARY YEARS 5 & 6

See Year 4 Vocabulary plus the following Year 5 specific vocabulary:

Ones boundary,

Tenths boundary

Multiplica–

Skill	Year Group(s)	Representations and models
Solve one-step problems with multiplication.	1/2	Bar model, Numicon, counters, tens frames, arrays.
Multiply 2-digit numbers by 1-digit numbers	3/4	Place value counters, Base 10, short written method, expanded written method.
Multiply 3-digit numbers by 1-digit numbers.	4	Place value counters, Base 10, short written method.
Multiply 4-digit numbers by 1-digit numbers.	5	Place value counters, short written method.
Multiply 2-digit numbers by 2-digit numbers.	5	Place value counters, Base 10, short written method, grid method.
Multiply 2-digit numbers by 3-digit numbers.	5	Place value counters, short written method, grid method.
Multiply 2-digit numbers by 4-digit numbers.	5/6	Formal written method.

Multiplica–

Before moving on to solve multiplication problems, children should become confident with doubling and practically counting in multiples.

CONCRETE (objects)

DOUBLES

Use practical activities to model how to double a number.

E.g cubes, counters, pencils, people etc.

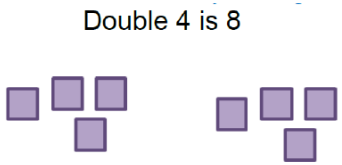
COUNTING IN MULTIPLES

Count in multiples supported by concrete objects in equal groups.

PICTORIAL

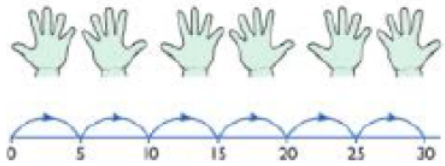
DOUBLES

Draw pictures to show how to double a number.



COUNTING IN MULTIPLES

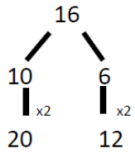
Use a number line or pictures to continue support in counting in multiples.



ABSTRACT

DOUBLES

Partition the number and then double each part before recombining it back together.



COUNTING IN MULTIPLES

Count in multiples of a number out loud.

Write sequences with multiples of numbers.

2, 4, 6, 8, 10

5, 10, 15, 20, 25

Skill: Doubling numbers and counting in multiples.

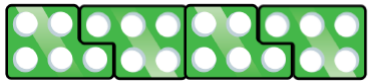
Years 1 & 2

Multiplica–

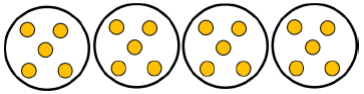
Children represent multiplication as repeated addition in many different ways. In Year 1, children use concrete and pictorial representations to solve problems. In Year 2, children are expected to record multiplication formally as number sentences.

CONCRETE (objects)

Children use Numicon (with support) to get the right amount of groups and then count the total in multiples.



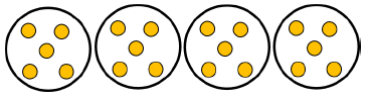
They then move onto moving counters into groups.



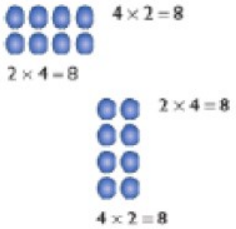
Children then move these counters into arrays to solve multiplication number sentences.

PICTORIAL

In Year 1, children draw the number of groups (large circles) and then draw the correct number of counters in each group.



In Year 2, children draw arrays in different rotations to find **commutative** multiplication sentences.



ABSTRACT

In Year 1, children write addition number sentences to match objects or pictures

In Year 2, children use an array to write multiplication sentences.

5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

5 x 3 = 15

3 x 5 = 15

Skill: Solve one-step problems with multiplication

Years 1 & 2

Multiplica–

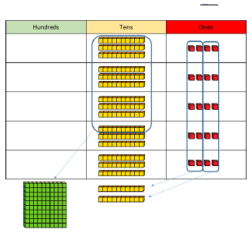
Children will look at the expanded column method before moving onto the short multiplication method. The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as the children should use times table knowledge to support their learning.

CONCRETE (objects)

Expanded column method

34 X 5 = 170

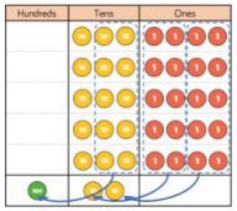
Children begin using Base 10 to get 5 groups of 34. They then count the ones and tens and exchange 10 ones for a 10s stick and 10 tens sticks for a hundred.



Then they can count the Base 10 to solve the answer to the calculation.

Short Multiplication Method

During the concrete stage this method follows the same rules as above but can be solved using place value counters.



PICTORIAL

Children can draw a pictorial representation of the concrete method. This is a 'stepping stone' method and can initially be used alongside the abstract method to support understanding.

ABSTRACT

Expanded column method

Children begin by writing the calculation. Underneath this, they write the two number sentences that they will need to solve and write the answers. They then use column addition to find the answer.

	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

Short Multiplication Method

This method directly follows on from the expanded column method, Children will solve the multiplication number sentences mentally.

E.g. 5 X 4 = 20 (carry the 2)

5 X 3 = 15 (add the 2 that you carried)

	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		

Skill: Multiply 2-digit numbers by 1-digit numbers.

Years 3 & 4

Multiplica–

Children should be encouraged to move towards the short, formal written method. Base 10 and place value counters can be used to support during the concrete phase.

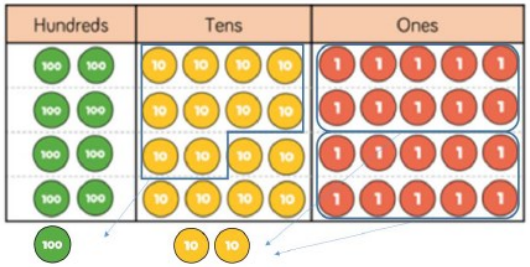
CONCRETE (objects)

Short column method

295 x 4 = 980

Children begin using Base 10 or place value counters. They then count the ones and tens and exchange 10 ones for a 10s stick and 10 tens sticks for a hundred.

Then they can count the Base 10 or counters to solve the answer



PICTORIAL

Children can draw a pictorial representation of the concrete method. This is a 'stepping stone' method and it can initially be used alongside the abstract method to support understanding.

ABSTRACT

Short Multiplication Method

This method directly follows on from the expanded column method, Children will solve the multiplication number sentences mentally.

For example,

295 x 4 = 980

5 x 4 = 20 (carry the 2)

4 x 4 = 16 + 2 (that you carried) = 18 (carry the 1)

4 x 2 = 8 + 1 (that you carried) = 9

	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

Skill: Multiply 3-digit numbers by 1-digit numbers.

Year 4

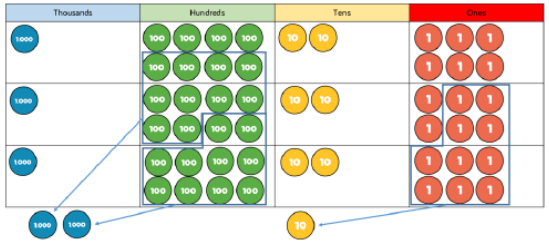
Multiplica–

When multiplying 4-digit numbers, place value counters are the best resource that children can use to support their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage them to use a multiplication grid so that they can focus on the method.

CONCRETE (objects)

1,826 x 3 = 5,478

Children begin using place value counters. They partition the number into thousands, hundreds, tens and ones. They then make any required exchanges. Finally, they add up the ones, tens, hundreds and thousands to solve the calculation.



PICTORIAL

Children can draw a pictorial representation of the concrete method. This is a 'stepping stone' method and can it initially be used alongside the abstract method to support understanding.

ABSTRACT

Formal Method

This method directly follows on from the expanded column method, Children will solve the multiplication number sentences mentally.

1,826 x 3 = 5,478

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

Skill: Multiply 4-digit numbers by 1-digit numbers.

Year 5

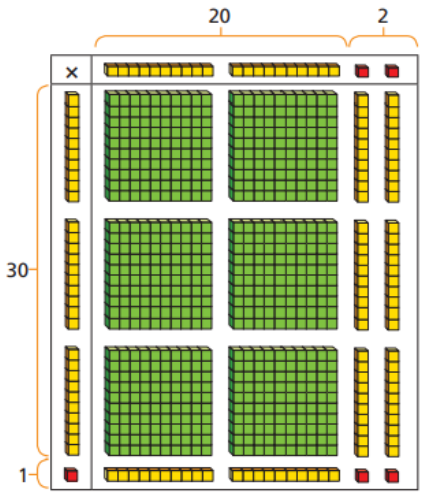
Multiplica–

When multiplying 2-digit numbers by 2-digit numbers. The area grid method can be used to help children understand the size of the numbers they are working with. The grid method then matches the area grid method and it is an initial written method.

CONCRETE (objects)

22 x 31 = 682

Partition both 2-digit numbers and then multiply them together.



The Base 10 is useful to help children think about the size of the numbers they are working with.

PICTORIAL

Following on from the area grid method, children will move towards using the grid method.

This method is much more efficient yet it is still a more 'visual' method than the formal method.

22 x 31 = 682

×	20	2
30	600	60
1	20	2

ABSTRACT

Formal Written Method

This method follows on from the grid method. Children will solve the multiplication number sentences mentally.

22 x 31 = 682

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

Children should be remind that they need to use a place holder and understand why this is the case.

Skill: Multiplying 2-digit numbers by 2-digit numbers.

Multiply 2-digit by 3-digit numbers

Year 5

Multiplica–

When multiplying 4-digit numbers by 2-digit numbers the children should already be confident in using the formal written method.

ABSTRACT

Formal Written Method

The layout for this needs to be taught explicitly as it can lead to errors being made. Children should always use squared paper when laying out calculations.

Considerations should be made for where exchanged digits are places and ensure that this is consistent.

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 (2)	1 (5)	9 (3)	1 (7)	2
5 (1)	4	7 (1)	8	0
7	6	6	9	2
		1		

Skill: Multiply 4-digit numbers by 2-digit numbers.

Year 5 & 6

Vocabulary: Multiplica–

KEY VOCABULARY YEAR 1

Multiplication

Multiply

Multiplied by

Times

Doubling

Array

Number patterns

Groups of

KEY VOCABULARY YEAR 2

See Year 1 Vocabulary plus the following Y2 specific vocabulary:

Once, twice, three times...tens times

Repeated addition

Equal groups of

Row

Column

Multiplication table

Multiplication fact

KEY VOCABULARY YEAR 3

See Year 2 Vocabulary plus the following Y3 specific vocabulary:

Short written method

Expanded written method

Factor

Product

Remainder

KEY VOCABULARY YEAR 4

See Year 3 Vocabulary plus the following Year 4 specific vocabulary:

Inverse, square, cube, cubed

KEY VOCABULARY YEARS 5 & 6

See Year 4 Vocabulary plus the following Year 5 specific vocabulary:

Formal written method

Grid method

Division

Skill	Year Group(s)	Representations and models
Solve one-step problems with division.	1/2	Bar model, objects, arrays, counters, Numicon, number lines.
Divide 2-digit numbers by 1-digit (no exchange sharing)	3	Straws, Base 10, bar model, place value counters, part-whole model.
Divide 2-digit numbers by 1-digit numbers (sharing with remainders)	3/4	Objects, place value counters, counters, number line, part-whole model.
Divide 2-digit numbers by 1-digit (grouping)	4/5	Place value counters, counters, place value grid, short division.
Divide 3-digits numbers by 1 -digit (sharing with ex-	4	Base 10, bar model, place value counters, part-whole model.
Divide 3-digit numbers by 1-digit (grouping)	4/5	Place value counters, counters, place value grid, short division.
Divide 4-digit numbers by 1-digit (grouping)	5	Place value counters, counters, place value grid, short division.
Divide multi-digit numbers by 2-digits (short division)	6	Written long division, list of multiples.
Divide multi-digit numbers by 2-digits (long division)	6	Written long division, list of multiples.

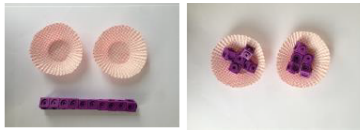
Division

To begin with, children should solve problems by sharing amounts into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. In Year 2, children are expected to use a more abstract approach and record their own number sentences.

CONCRETE (objects)

SHARING INTO GROUPS

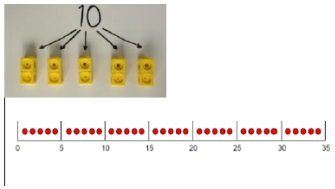
Use objects to share amounts into equal groups. E.g. I have ten cubes, can you share them into two equal groups?



DIVISION AS GROUPING

Divide quantities into equal groups.

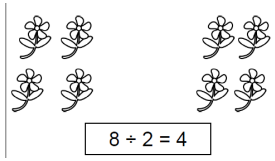
Use cubes, counters or any other objects to support understanding.



PICTORIAL

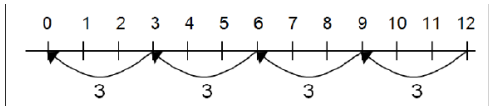
SHARING INTO GROUPS

Children use pictures or shapes to share quantities equally.

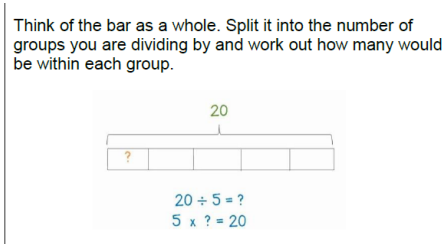


DIVISION AS GROUPING

Use a number line to show jumps in groups. The number of jumps equals the number of groups.



Also use the bar model to show number of groups within a number as a total.



ABSTRACT

SHARING INTO GROUPS

Solving problems and using the correct symbols to show division by sharing.

Share 9 buns between three people.

9 ÷ 3 = 3

DIVISION AS GROUPING

Solving problems and using the correct symbols to show division by grouping.

28 ÷ 7 = 4

Divide 28 into 7 groups.
How many are in each group?

Skill: Solve one-step problems with division.

Years 1 & 2

Division

CONCRETE (objects)

DIVISION WITIHIN ARRAYS (INVERSE)

Link division to multiplication by creating and array and thinking about the number sentences that can be made from it.

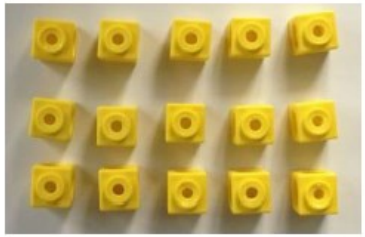
E.g.

3 x 5 = 15

5 x 3 = 15

15 ÷ 3 = 5

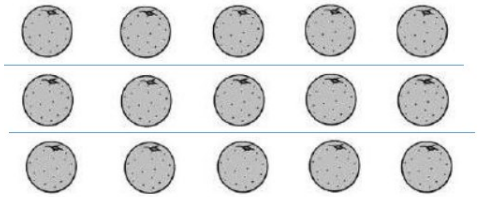
15 ÷ 5 = 3



PICTORIAL

DIVISION WITIHIN ARRAYS (INVERSE)

Draw an array and use lines to split the array into groups to make multiplication and division sentences.



ABSTRACT

DIVISION WITIHIN ARRAYS (INVERSE)

Find the inverse of multiplication and division sentences by finding all 4 possibilities.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7

Skill: Solve one-step problems with division.

Years 1 & 2

Division

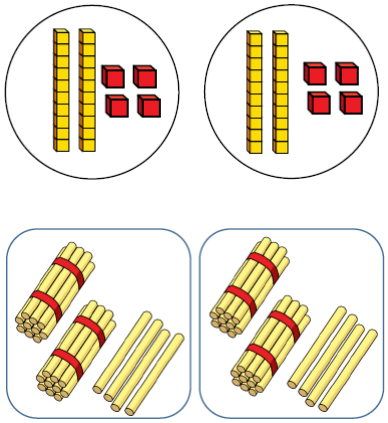
When dividing larger numbers, children can use concrete objects to allow them to partition numbers into tens and ones.

CONCRETE (objects)

Children can use straws and Base 10 to create groups of tens and ones. They can then share these equally into the correct number of groups.

For example,

48 ÷ 2 = 24

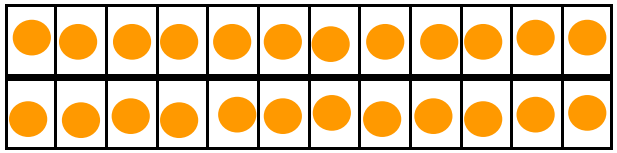


PICTORIAL

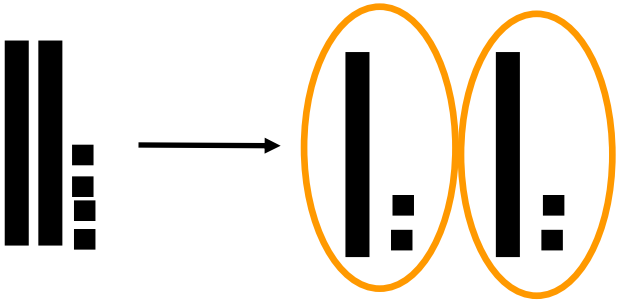
Once children are confident using the physical Base 10 resource they can then move onto drawing jottings or arrays.

24 ÷ 2 = 12

With an array



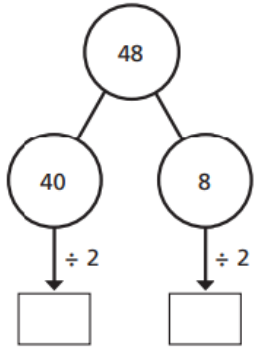
With Base 10 jottings



ABSTRACT

To support children to solve the calculation without pictorial or concrete representations children can use a part-whole model.

Children partition the number they are dividing and then divide those numbers before combing them back together at the end to get the answer.



Skill: Divide 2-digits by 1-digit (sharing with no exchange)

Year 3

Division

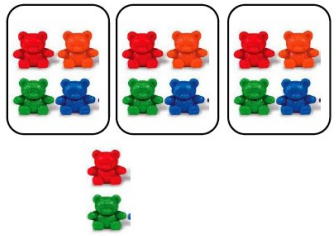
When dividing numbers with remainders, children can use Base 10, objects and place value counters to support children's initial understanding. Starting with equipment outside of the grids allows them to see the remainders more easily.

CONCRETE (objects)

DIVISION WITH A REMAINDER

Divide objects between groups and see how much is left over.

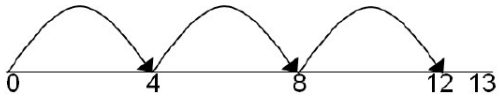
14 ÷3 = 4 r 2



PICTORIAL

DIVISION WITH A REMAINDER

Jump forward in equal jumps on a number line and then see how many more you need to find the remainder.

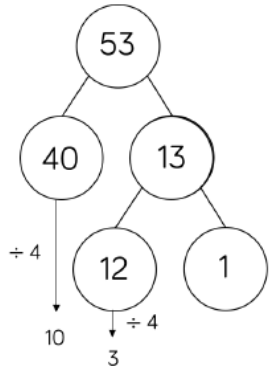


Alternatively or in addition to, draw dots and group them to divide an amount and clearly show a remainder.

ABSTRACT

DIVISION WITH A REMAINDER

As with the previous skill, children can use a part-whole model to partition the number they are dividing. Then they can divide those numbers and ascertain how many remainders there are.



Complete written divisions and show the remainder using 'r'.

Skill: Divide 2-digits by 1-digit numbers (sharing with remainders)

Years 3 & 4

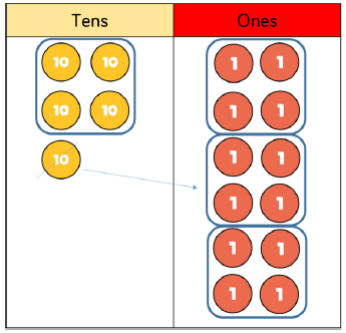
Division

When using the short division method, children use grouping. Starting with the largest place value, they groups by the divisor. Using the correct vocabulary is really important. "How many groups of 4 tens can we make?"

CONCRETE (objects)

Using place value counters to 'group' is the first step towards using the short method for division.

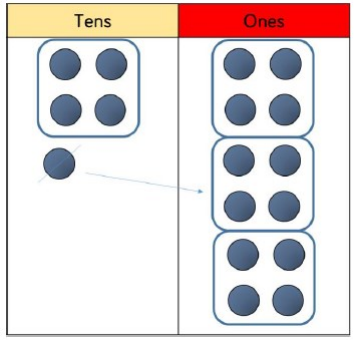
52 ÷ 4 = 13



PICTORIAL

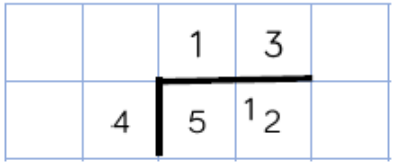
The next step is to draw 'counters' into a place value grid and make annotations. This follows the concrete method very closely.

52 ÷ 4 = 13



ABSTRACT

Begin with divisions where there are no remainders.



Children start with the tens.

How many times does 4 go into 5? 1 with 1 remainder. Write this remainder next to the ones digit.

How many times does 4 go into 12? 3 with no remainders.

Skill: Divide 2-digits by 1-digit (grouping).

Years 4 & 5

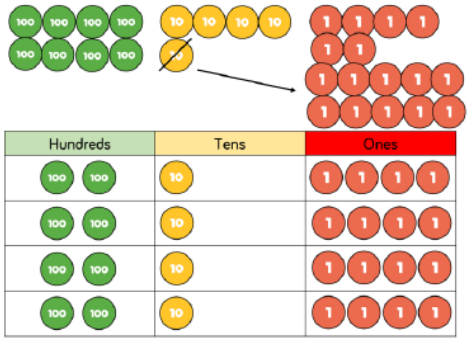
Division

Children should continue to use place value counters to divide 3-digit numbers. They should start with equipment outside of the grids as this allows remainders to be clearly highlighted.

CONCRETE (objects)

Children to make the 3-digit number that they are sharing with place value counters (or Base 10). They place this number outside of a place value grid. They then share the hundreds, tens and ones. If there are any hundreds or tens left over they can be exchanged. If there are any ones left over they are the remainders.

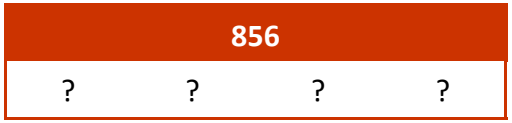
856 ÷ 4 = 214



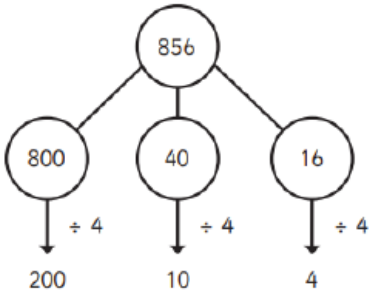
PICTORIAL

Children are to begin by drawing a bar model. This allows them to visualise the calculation.

856 ÷ 4 = 214

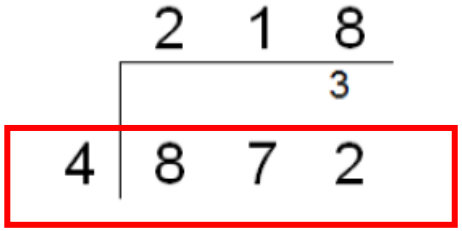


After this, they can complete a part-whole model.



ABSTRACT

Begin with divisions where there are no remainders.



I start by thinking, does 4 go into 8? Yes, two times.

Write 2 on top of the line.

Now does 4 go into 7? Only once and there are 3 left over so write 1 and carry the 3 over to the 2.

Does 4 go into 32? Yes, 4 8 times so I write 8 above the line.

The answer is 218.

Skill: Divide 3-digits by 1-digit (sharing with exchange)

Year 4

Division

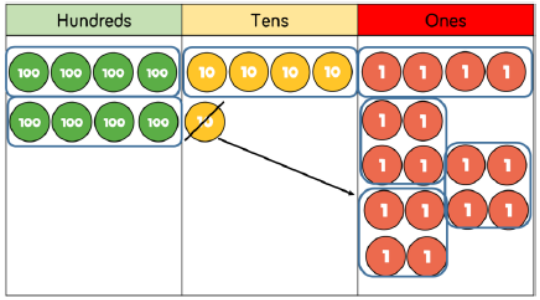
Children can continue to use grouping to support their understanding of short division.

CONCRETE (objects)

Children to make the 3-digit number that they are sharing with place value counters (or Base 10).

They partition the number they are dividing into the grid. They then draw groups around the number they are dividing by.

856 ÷ 4 = 214

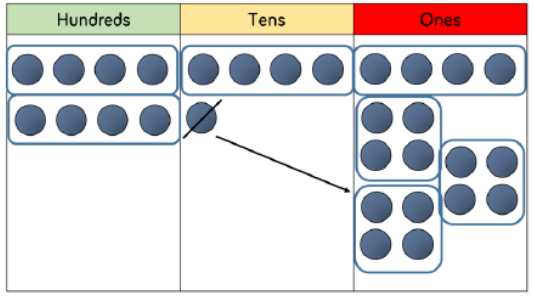


PICTORIAL

Children to draw the 3-digit number that they are sharing into a place value grid.

They partition the number they are dividing into the grid. They then draw groups around the number they are dividing by.

856 ÷ 4 = 214



ABSTRACT

Begin with divisions where there are no remainders. Children start with grouping the hundreds, then tens and then finally the ones digit.

		2	1	4
	4	8	5	$^{1}6$

Skill: Divide 3-digits by 1-digit (grouping)

Years 4 & 5

Division

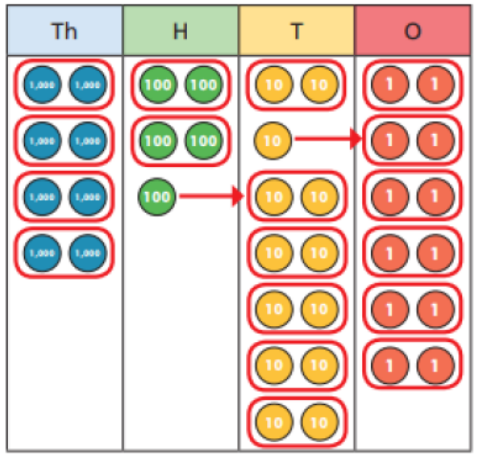
Children can continue to use grouping to support their understanding of short division.

CONCRETE (objects)

Children to partition the 4-digit number that they are sharing with place value counters (or Base 10).

They partition the number they are dividing into a place value grid. They then draw groups around the number they are dividing by.

8,532 ÷ 2 = 4,266



PICTORIAL

Children to draw the 4-digit number that they are sharing into a place value grid.

They partition the number they are dividing into the grid. They then draw groups around the number they are dividing by.

This step will mirror the concrete approach but it will use jottings/drawings rather than the physical objects.

ABSTRACT

Begin with divisions where there are no remainders. Children start with grouping the thousands, then the hundreds, tens and then finally the ones digit.

	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

Skill: Divide 4-diigits by 1-digit (grouping)

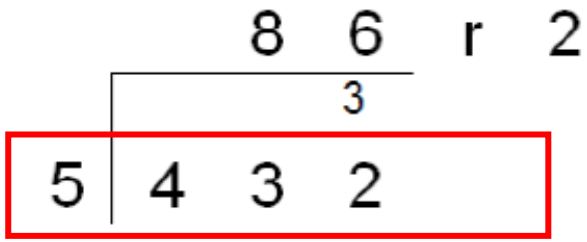
Year 5

Division

When children begin to divide up to 4-digit numbers by 2-digits, written methods become the most accurate and effective method.

ABSTRACT

Begin with divisions where there are no remainders.



I start by thinking, does 5 go into 4? No

Carry the 4 over to the 3 to create 43.

Now does 5 go into 43? Yes, 8 times with 3 leftover.

Write the 8 above the line and carry over the 3.

Does 5 go into 32? Yes, 6 times with a remainder of 2.

The answer is 86 r2.

7,335 ÷ 15 = 489

	0	4	8	9
15	7	7 3	13 3	13 5

15	30	45	60	75	90	105	120	135	150

Skill: Divide multi-digits by 2-digits (short division)

Year 6

Division

ABSTRACT

Start with calculations that have no remainders.

432 ÷ 12 = 36

To support their use of this method children should write down the multiplication table at the side of the method.

'How many 12s go into 432?' 12 x **30** = 360.

432– 360 = 72.

'How many 12s go into 72?' 12 x **6** = 72

72—72 = 0.

30 + 6 = 36

		0	3	6	
1	2	4	3	2	(×30)
	–	3	6	0	
			7	2	(×6)
	–		7	2	
				0	

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

7,335 ÷ 15 = 489

'How many 15s go into 7,335?' 15 x **400** = 6,000

7,335—6000 = 1335.

'How many 15s go into 1335?' 15 x **80** = 1,200

1335—1200 = 135.

'How many 15s go into 135?' 15 x **9** = 135

135—135 = 0

400 + 80 + 9 = 489.

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Skill: Divide multi-digits by 2-digits (long division)

Year 6

Vocabulary: Division

KEY VOCABULARY YEAR 1

Share

Sharing

Division

Dividing

Halving

Array

Number patterns

Groups

KEY VOCABULARY YEAR 2

See Year 1 Vocabulary plus the following Y2 specific vocabulary:

Once, twice, three times...tens times

Repeated subtraction

Divide

Divided by

Divided into

Share equally

Left over

Division fact

KEY VOCABULARY YEAR 3

See Year 2 Vocabulary plus the following Y3 specific vocabulary:

Short written method

Expanded written method

Factor

Product

Remainder

KEY VOCABULARY YEAR 4

See Year 3 Vocabulary plus the following Year 4 specific vocabulary:

Inverse, square, cube, cubed

KEY VOCABULARY YEARS 5 & 6

See Year 4 Vocabulary plus the following Year 5 specific vocabulary:

Formal written method

Grid method